Utilising convolutions of random functions to realise function calculation via a physical channel

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***Abstract*—We discuss the utilisation of an algebra of random functions for the calculation of mathematical operations on a physical communication channel for actual implementation with resource restricted nodes. In particular, we present a transmission scheme for the computation of functions on the wireless channel and discuss various properties from combinations of random functions as well as requirements and restrictions of resource restricted hardware.**

***Index Terms*—Physical-layer function computation, Wireless communication, Cooperative systems, Statistical distributions**

I. INTRODUCTION

In recent years, we have witnessed a technological evolution in which electronic devices are becoming more widespread, more powerful and significantly smaller. Nowadays, communication and computation capabilities are inherent in many devices in our environment. This progress will continue and lead to an Internet of Things (IoT). In an IoT we expect a high penetration of resource limited distributed devices that incorporate an interface to the RF-channel. In order to empower these devices to calculate ambitious mathematical operations despite their sharp resource restriction we propose to employ the physical channel as a calculator for specific operations. In particular, we envision a network of distributed nodes offloading part of a computation solved collaboratively to the wireless channel. By simultaneously transmitting values v_i from n nodes over the wireless channel, a receiver will observe a superimposition

$$\zeta_{\text{rec}}(t) = \Re\left[\left(e^{j2\pi f_c t}\sum_{i=1}^{n} v_i \text{RSS}_i e^{j\gamma_i}\right) + \zeta_{\text{noise}}(t)\right] \quad (1)$$

of these values. In equation (1), f_c represents a common transmit frequency, RSS_i and γ_i the received signal strength and relative phase shift of a signal component transmitted by node i and $\zeta_{\text{noise}}(t)$ the noise signal. We understand the simultaneous transmission over a wireless channel as the execution of a function $y = f(v_1, \ldots, v_n)$ with the transmitted values v_i as input parameters and the reception $\zeta_{\text{rec}}(t)$ as the output y of the function.

Recently, the feasibility of calculating functions on the wireless channel has been considered by various authors [1], [2], [3]. These approaches require a tight synchronisation of transmitted superimposed signals on the symbol or phase level or regarding their transmit energy. This, however, is hard to establish for resource limited IoT nodes. As an alternative, robust against inaccurate power, phase or symbol synchronisation, we utilise in [4], [5] Poisson distributed burst sequences to realise computation on the wireless channel. In particular, we could compute all four basic mathematical operations.

In this paper we detail the underlying operational principle and discuss the utilisation of other probability distributions for the realisation of further mathematical operations. The following sections are structured as follows. Section II summarises the related work on function computation via the RF-channel. In section III we then present the scheme for the calculation utilising an algebra of random functions (section III-A), calculate the expected error in a computation (section III-B) and detail functions computable by various probability distributions (section III-C). Section IV then discusses practical aspects for an actual implementation of channel-supported mathematical operations and section V draws our conclusion.

II. RELATED WORK

Distributed computation of functions over a set of connected nodes is traditionally achieved by (possibly multiple times) aggregating values from remote devices at a sink node which then calculates the desired operation on the aggregated data [6]. In this computational scheme, however, since a single node conducts the final computation, the computational load among nodes is heavily unbalanced. Recently, several authors have proposed to outsource this final computation to the wireless channel as a function automatically computed during the superimposition of simultaneously transmitted information.

For instance, in [1], Nazer and others consider the problem of computing a function of a set of inputs from distributed nodes over a multiple access channel. They develop schemes that outperform channels based on the separation of communication and computation. In particular, they study the achievable rate for reliably implementing arbitrary functions over multiple access channels. This work provides the information theoretic background for function computation over a wireless channel.

Another approach for function computation on the wireless channel was presented by Keller and others [3]. The authors present communication protocols for channel-based computation by utilising appropriate encodings of symbols to be combined on the channel. They derive functions computable in a single or multiple time slots and further show that by using an interactive protocol, more efficient schemes are possible. These approaches require accurate synchronisation of nodes at the symbol or phase level which is hard to achieve with strongly resource restricted IoT nodes.

Goldenbaum and others overcome this limitation by exploiting the signal strength of a received superimposition of signals [2], [7], [8]. They show how the arithmetic mean, the geometric mean, polynomials and other functions can be calculated at the time of transmission on the wireless channel. They utilise superimposition of electromagnetic signals to realise a sum of various input values and pre- and post-processing at transmit and receive nodes in order to compute more ambitious functions. Their solution requires accurate synchronisation among transmit nodes regarding their transmit power which is hard to achieve for resource limited IoT nodes.

In a related study in [4], [5] we proposed a transmission scheme utilising Poisson-distributed burst sequences to calculate mathematical functions during superimposition of transmitted values on a wireless channel. The scheme is tolerant for inaccurate synchronisation of nodes with respect to phase, transmit symbols or transmission power and was implemented and demonstrated on single sensor nodes.

In [9], [10] we have presented an approach to compute a distribution of values transmitted simultaneously by an arbitrary number of unsynchronised sensor nodes as random binary sequences. In a related work, we utilised of beamforming to generate a virtual neural-network overlay on top of a network of distributed nodes [11].

Another related area is the analog or physical layer network coding. In these approaches, although no mathematical function is computed on the wireless channel, superimpositions of simultaneous transmissions are utilised to combine values from distinct nodes on the physical layer. Analog network coding combines signals on the physical channel and forwards these superimpositions to a destination that exploits network-level information to extract the desired signal [12]. For physical layer network coding, Zhang and others present an approach in which binary transmit sequences of electromagnetic signals are combined on the wireless channel during simultaneous transmission [13]. Nazer and Gastpar summarise recent progress in physical layer network coding and utilise linear error correcting codes to allow nodes to recover linear combinations derived from noisy observations [14].

III. Calculation of functions on the RF channel

Current schemes proposed for the calculation of functions on the wireless channel have two main drawbacks. Either transmissions have to be tightly synchronised on the symbol and phase level [1] or transmitters are required to synchronise their received signal strength [2]. Both these requirements are hard to achieve for simple nodes.

We propose a transmission scheme utilising random burst sequences in order to mitigate both disadvantages at the cost of an increased time to complete a calculation in section III-A, investigate the expected error in section III-B and discuss possible realisations of mathematical functions in section III-C. Since sequences of bursts are utilised, the scheme is less susceptible to fading.

A. Scheme

We utilise superimposed burst sequences from various transmitters for the calculation of functions. The general principle is briefly sketched in figure 1. Burst sequences from various

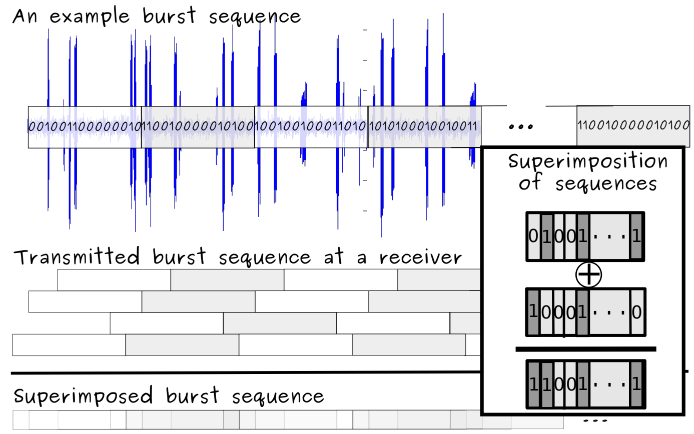

Fig. 1: A sketch of the scheme for the computation of mathematical functions via a communication channel

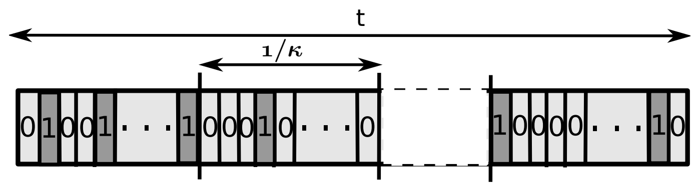

Fig. 2: A concatenated burst sequence

transmit sources are overlayed at a receiver. During this process, bursts write though into the received burst sequence. The underlying physical principle for the calculation scheme is therefore the cumulation of bursts in a received sequence. Different functions can be realised by the differing interpretations of these bursts. For instance, a naive implementation of an addition on the wireless channel could simply count the number of bursts in a received sequence in order to estimate the overall count of bursts in all transmitted sequences. In order to mitigate the impact of collisions on the derived value, we encode a transmitted value repeatedly in multiple concatenated sub-sequences.

Before we consider concrete implementations, we will discuss the robustness of these transmission scheme towards inaccurate synchronisation in phase, symbol or signal strength level.

Since burst sequences are utilised, the exact received power can fluctuate greatly among sequences, provided that the burst sequence received with least power is still significantly above the noise level. Also, burst sequences do not require exact synchronisation in transmit phases.

In order to understand the impact of inaccurate synchronisation on symbol (burst) level, we assume an encoding of a specific value v κt times in a sequence of length t that is concatenated from sequences of length $\frac{1}{\kappa}$ (cf. figure 2). We will utilise representations of values as random burst sequences with random distributions which are tolerant for shift and reordering of bursts over a sequence. Furthermore, we implement a value in this manner several times within an overall burst sequence in concatenated sub-sequences. Consequently, taking these two preconditions together, regardless of which distinct $\frac{1}{k}$ bits from the received sequence are considered by the receiver, they will always encode the same random distribution with about the same properties. Slight variation for a different choice of $\frac{1}{k}$ bits then result only from possible collisions in burst sequences. Consequently, sequences do not have to be synchronised on the symbol level.

B. Calculation of errors induced by collisions

The amount of collisions in a received burst sequence impacts the error in the described calculation. With accurate knowledge on the expected amount of collisions it is therefore possible to correct an error in a calculation on a wireless channel to some extent. In the following, we calculate the probability of collisions and their expectation.

We assume two sequences V_1 and V_2 of length t with v_1, respective v_2 bursts. The probability of i collisions between these sequences is then described as

$$\underbrace{\binom{v_2}{i}}_{\text{Allocations of } i \text{ collisions in } v_2 \text{ bursts of sequence } V_2} \cdot \prod_{j=1}^{i} \underbrace{\frac{v_2-j+1}{t-j+1}}_{\text{Probability to hit one of the remaining } v_2-j+1 \text{ bursts among the } n-j+1 \text{ positions}} + \underbrace{\left(1-\frac{v_2}{t}\right)^{v_1-i}}_{\text{Don't hit any other position}} \quad (2)$$

Consequently, we can state the probability of no collision as

$$1-\left(\sum_{i=1}^{v_1}\binom{v_2}{i}\cdot\prod_{j=1}^{i}\frac{v_2-j+1}{t-j+1}+\left(1-\frac{v_2}{t}\right)^{v_1-i}\right) \quad (3)$$

We calculate the expected number of collisions from two sequences therefore as

$$E[\text{Collisions}(v_1,v_2)] = \sum_{i=1}^{v_1} i\binom{v_2}{i}\prod_{j=1}^{i}\frac{v_2-j+1}{t-j+1}+\left(1-\frac{v_2}{t}\right)^{v_1-i} \quad (4)$$

Provided that more than two sequences are overlayed, we can calculate the probability of i collisions in the overall superimposed sequence by cumulating a sum of terms of the form given in equation (2) for each sequence above the first. The expected number of collisions in such a case is calculated accordingly.

C. Probability distributions

In the following we recapitulate various properties of different probability distributions and detail concrete implementations to conduct mathematical computations on a wireless channel.

1) Poisson distribution: In [4] and [5] we have presented a transmission scheme to realise all four basic mathematical operations using Poisson-distributed burst-sequences. The scheme is tolerant to weak synchronisation of nodes and requires only simple operations by participating nodes. The general principle grounds on the following property.

For two sequences encoding Poisson variables χ_1 and χ_2 with means μ_1 and μ_2, their combination $\chi_1+\chi_2$ again yields a Poisson distributed variable with mean $\mu_1+\mu_2$ [15].

We divide a burst sequence of length t into κt sub-sequences of length $\frac{1}{\kappa}$. Each of these subsequences contains with probability p_κ one or more of a finite number of bursts. The Poisson distribution then defines the probability to find k bursts in this sequence as [16]

$$p(k;\mu t) = e^{-\mu t}\frac{(\mu t)^k}{k!}. \quad (5)$$

The parameter μ determines the density of bursts within the sequence. The larger μ is, the smaller the probability of finding no point. It is also the mean of the distribution.

To transmit a value v_i we define a Poisson process with mean $\mu = v_i$. The transmit sequence is then designed such that each of the sub-intervals has the probability $p(k;\mu t)$ (cf. equation (5)) that it contains exactly k bursts. To observe the value μ at a receiver, we extract the count N_i of sub-sequences with exactly i bursts as well as the total number of bursts $T=\sum_{i=1}^{n} i\cdot N_i$. If $N=\sum_{i=1}^{n} N_i$ is large, we expect that $N_k \approx Np(k;\mu t)$ [16]. We can conclude

$$\begin{aligned} T &\approx N\left(p(1;\mu t)+2p(2;\mu t)+\ldots\right) \\ &= Ne^{-\mu t}\mu t\left(1+\frac{\mu t}{1}+\frac{(\mu t)^2)}{2!}+\ldots\right) \\ &= N\mu t \end{aligned} \quad (6)$$

and consequently

$$\mu t \approx \frac{T}{N}. \quad (7)$$

A receiver can therefore extract μ from a received sequence.

By utilising logarithm laws and fractions of values we can then generalise this method to solve all four (addition, subtraction, multiplication, division) basic mathematical operations on the wireless channel.

2) Geometric distribution: For calculating the product of values, we might also utilise an interesting property of geometric distributed random variables. If χ_1 and χ_2 are geometric random variables with probability of success p_1 and p_2 respectively, then $\min(\chi_1,\chi_2)$ is a geometric random variable with probability of success $p = p_1+p_2-p_1\cdot p_2$. More straightforward and easy to use is the relationship in terms of failure probabilities: $q = q_1 q_2$.

The geometric distribution describes the number of Bernoulli trials necessary for a discrete process to change state. In order to multiply two values v_1 and v_2 we might therefore choose geometrically distributed random processes with failure probabilities q_1 and q_2. For each of the concatenated subsequences of length $\frac{1}{\kappa}$ we will from left to right conduct Bernoulli trials until the first trial was successful. A single burst is transmitted only at the position where the first successful trial was conducted.

Figure 3 illustrates this scheme. When now two sequences, synchronised at burst-level, are superimposed, the resulting sequence will feature one or two bursts in each sub-sequence. At the receiver, we estimate the minimum value by considering

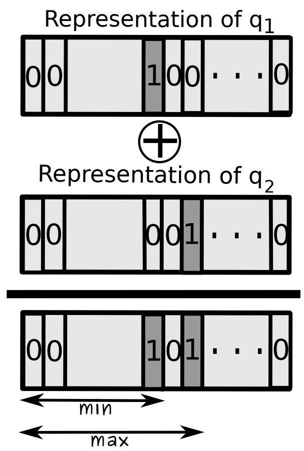

Fig. 3: Multiplication of values by superimposition of burst sequences with an underlying geometric distribution

the leftmost burst in each sub-sequence. The distribution is estimated from the observed superimposed sequence in similar manner as described for the Poisson distribution.

3) Exponential distribution: With some combinations of distributions, it is also possible to apply specific more complex operations on the wireless channel. Consider, for instance, the combination of exponential random variables. If χ_1 and χ_2 are exponential random variables with mean μ_1 and μ_2 respectively, then $\min(\chi_1, \chi_2)$ is an exponential random variable with mean $\frac{\mu_1\mu_2}{\mu_1+\mu_2}$.

The exponential distribution describes the time for a continuous process to change state. In particular, it can be seen as the continuous counterpart of the geometric distribution. For instance, the exponential distribution could describe the time between two distinct Poisson events. The waiting time for a Poisson distribution is an exponential distribution with parameter μ. Therefore, we can again use Poisson distributed sequences in order to represent an exponential distribution. However, for the channel to calculate the minimum of two variables μ_1 and μ_2 we propose to encode the time for the process to change state as sequence of leading zeros. By implementing a value as burst-sequence of leading zeros, a synchronised superimposition of several of such sequences would lead to an encoding of the the minimum of these sequences. The implementation can thus be realised in the same manner as the encoding illustrated in figure 3). Hence, a single node $\mathcal{N}_i$ willing to transmit a value v_i will repeatedly encode results of random experiments with a geometric distributed random variable χ_1 with mean μ_1 and combine this as binary sequence of leading zeros to a concatenated sequence S_1. When this is superimposed in a synchronised manner with a second sequence S_2, encoding a value v_2, a receiver may then calculate observe a exponential distributed process with mean $\frac{\mu_1\mu_2}{\mu_1+\mu_2}$ from the superimposition of S_1 and S_2.

IV. Practical issues

In order to realise mathematical operations on the wireless channel, specific nodes that support the described operations are required. We discuss practical issues encountered with traditional Si-based CMOS technology and emerging organic and printed electronics.

In the light of developing ultra-low-cost sensing devices such as Smart Labels based on organic and printed electronics [17], and on the other hand sensor-based RFID networks [18] produced in traditional Si-based technology for monitoring environments and tracking objects, the circuitry design of such devices requires, due to manufacturing costs and low power constraints, complete simpleness and extreme low hardware complexity. For instance, organic electronics (OE) promise to become easy to manufacture and low-cost technology, enabling item-level tagging. But, on the other hand, the surface consumption to integrate sufficient number of transistors to realise a digital circuit is too high, so that current state-of-the-art organic printed electronics is not able to realize IoT sensor nodes with standard communication protocols. While future technological advances might mitigate these hindrances, another motivation to consider highly resource restricted nodes is to enable self-contained wireless sensor networks relying entirely on power harvesting techniques [19]. An alternative until OE are more matured is SI-based CMOS hardware which provides high performance in terms of transmission range, carrier frequency generating, computational power, clocking frequency and surface consumption. However, the production costs for these devices are higher.

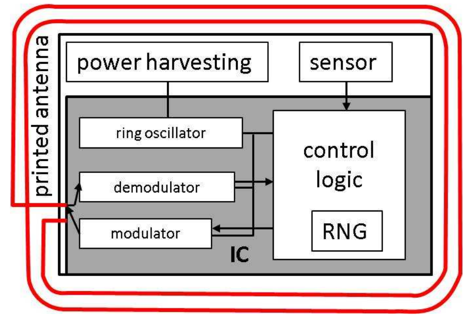

Fig. 4: System architecture of the resource restricted IoT sensor node. The random number generator (RNG) is an inherent part of the circuit logic.

The system architecture of the proposed resource restricted IoT node is shown in figure 4, consisting essentially of an integrated circuit (IC) and three external components such as the power harvesting unit, the sensor interface and the printed antenna on the circuit board. The circuit layout inside the IC is partitioned into several circuit functions such as the ring oscillator providing the synchronous system clock, the demodulator and modulator responsible for the external communication, and the control logic unit realizing the operating system of the sensor node.

A. Operation mode

In general the design goals of the resource restricted IoT node are based on providing a light weighted circuit design and enabling independent power supply for the sensor device.

For such energy harvesting, various forms exist, relying on thermal, solar, wind, vibration and electromagnetic RF waves. In case of receiving a request from a base station the demodulator of the sensor node converts the incoming signal into a digital code sequence, which activates the control logic to process the request. Based on the received command sequence, different procedures can be carried out such as the read out of the sensor and the translation it into a transmit sequence of bursts broadcast by the modulator.

B. Control logic

The operating system of the sensor node is realized in the control logic circuitry containing some finite state machines and a random number generator (RNG). In case of querying a sensor value, the control unit fetches the 8-bit sensor value and generates a Poison distributed random sequence indicating when a burst signal is to be send. The random burst sequence reflects the sensor value. Right after the transmitted burst sequence the control logic goes back into the initial state to carry out the next incoming command.

C. Computation on the RF-channel

The regular procedure for calculating a desired function on the wireless channel, such as calculating the mean temperature value in a scattered sensor network, is initiated at the base station by broadcasting a request command to all sensor nodes. Consequently, all sensor nodes are transmitting their random burst sequence approximately at the same time. Additional synchronization is not required for the calculation utilising Poisson distributed values described above. Fro the other discussed realisations, a synchronisation at burst level is required. However, by expanding the length of the burst sequence or of the bursts itself the tolerance for inaccurate synchronisation is increased. At the base station the superimposed burst sequence is evaluated by counting the occurred bursts in a fixed time frame. Based on the mean value of the obtained histogram the final outcome of the calculated function on the RF-channel is acquired.

V. Conclusion

We have discussed the calculation of mathematical operations using simultaneous transmission of values on the wireless channel. We detailed a general transmission scheme and possible realisations of mathematical operations utilising Poisson, Exponential and Geometric distributed random distributions encoded into transmit burst sequences. Possible realisations of these transmission schemes in a minimalistic wireless sensor node for an IoT were sketched. Further work covers the realisation of further operations by utilising other properties of random distributions.

For instance, normal or lognormal distributions might be utilised. If χ_1 and χ_2 are lognormal random variables with parameters (μ_1, σ_1^2) and (μ_2, σ_2^2) respectively, then $\chi_1 \cdot \chi_2$ is a lognormal random variable with parameters $(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$. Furthermore, if χ_1 is a normal (μ_1, σ_1^2) random variable and χ_2 is a normal (μ_2, σ_2^2) random variable, then $\chi_1 + \chi_2$ is a normal $(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$ random variable [15].

Acknowledgment

The authors would like to acknowledge funding by a fellowship within the Postdoc-Programme of the German Academic Exchange Service (DAAD).

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